Causal Networks, blocking, and d-separation

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The problem of how to infer causal relations has held the imagination of philosophers and scientists for centuries. From the 18th century, when Hume published his induction problem (how can we conclude that A causes B at all?), to roughly the end of the 20th century very little progress was made in explicating the conditions under which causal inference is possible. This changed in the late 1980s with the realization that the study of multivariate systems (systems that contain more variables than just A and B) allows for stronger inferences. Indeed, under some conditions, causal inferences *can* be made from correlational data, if the investigator is willing to invest the relevant assumptions. Excellent work on this is the book *Causality* by Judea Pearl (2000), who is responsible for many of the ideas explained below. Unfortunately, that book is very hard to read for students (and for staff as well, for that matter). Hence, the current note offers a short summary of the main ideas.

It should be possible to follow from this note the basic scheme of reasoning in thinking about causal networks. However, I do not explicitly introduce the concepts of probability and (conditional) independence, assuming that most students have received an introduction in basic probability theory. There is another note on BlackBoard (CI.pdf) that gives a short explanation of conditional independence, and many treatments are available online as well; probably, you could also confer your first-year statistics book.

1. Causal Networks

Our starting point is a network consisting of nodes and directed edges (arrows) between them, without feedback loops. We understand a directed edge from one variable (e.g., smoking) to another (e.g., lung cancer) to be a causal relation. This means that, if we (or Nature) were to manipulate the cause (e.g., if you're a non-smoker, we get you to take up smoking) we would alter the probability of the effect (e.g., your probability of lung cancer would rise). An example of such a network is given in Figure 1. The network in Figure 1 is *directed*, because all arrows go from one node to another: There are no connections with double arrowheads (implying feedback) or without arrowheads (which means variables are just correlated for reasons outside the model). In addition, if you walk through the network by following the arrows, you never walk in a circle. This means that the network is *acyclic* (there are no cycles). For this reason, such a structure is called a Directed Acyclic Graph (DAG).

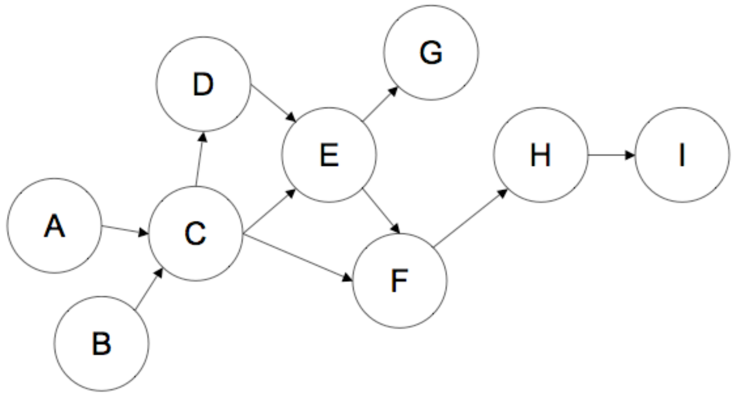


Figure 1. A Directed Acyclic Graph (DAG).

If we have just two variables, the only way to decisively reach a conclusion on what causes what is to use experimentation. However, if we have more than two variables, like in the graph above, we can derive testable hypotheses from a hypothesized causal structure. This means we can test whether a causal model fits the data, even though no experimental manipulation has been carried out. In rare but theoretically important cases, we can even reach decisive causal conclusions purely on the basis of correlational data. In addition, we can figure out which causal models could have generated the data, which means we can search for candidate models. This is because different models imply different patterns of conditional independence among variables.

2. Causal Structures and Conditional Independence

Consider a triples of variables A, B, and C. There are three important causal structures that these variables can form: Common cause structures, Collider structures, and Chains. In a common cause structure, one node causes the other nodes (B <- A -> C). If that is true, controlling for the common cause (A), renders the effects (B and C) independent. For instance, there is a positive correlation between the number of storks ("ooievaars"; B) and the number of babies born (C) across villages in Macedonia. This is not because storks bring babies, but because larger villages attract more storks and hold more people who produce more babies. Hence, if we control for the size of the villages (the common cause, A), then the correlation between storks and babies vanishes. This is called *conditional independence*: conditional on one variable (A), the other variables become uncorrelated (independent). If you do not know what conditional independence is, please consult your statistics books or the document "CI.pdf" on Blackboard, which contains a quick explanation.

In a chain (for instance, A -> B -> C), there is also conditional independence given the middle node (so, in this case A and C are conditionally independent given B). This also makes sense. For instance, if smoking (A) causes tar in the lungs (B) which causes cancer (C), then controlling for tar in the lungs interrupts the mechanism, and as a result cancer and smoking become uncorrelated. These structures are represented in Figure 2 below.

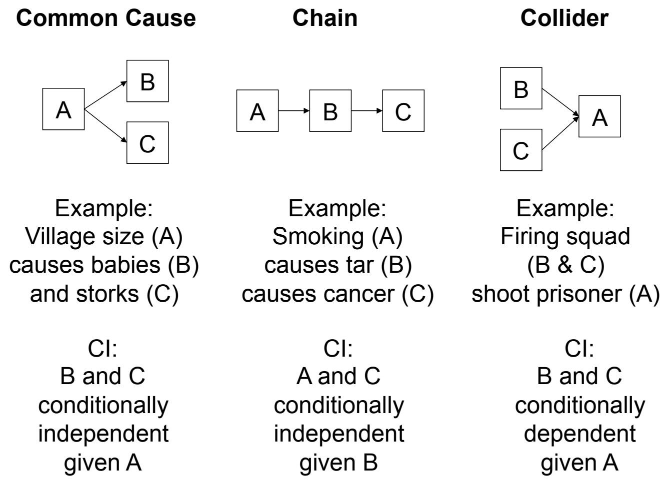


Figure 2. Three important causal structures and the associated patterns of conditional independence (CI).

The final structure is a collider structure, depicted in the right panel of Figure 2. In this structure, two variables jointly cause a third (A -> B <-C). In this case, you get conditional *dependence*. Even if the variables A and C are uncorrelated to begin with, once we condition on their common effect we create a correlation. For instance, suppose that two soldiers in a firing squad (B and C) shoot a prisoner (A). Suppose B and C fire completely independently, so that learning that B fired gives no information on whether C fired: B and C are *uncorrelated*. So suppose you know that B didn't fire. Then you have no information on whether C fired. However, now you learn that the prisoner is dead (conditioning on A). Suddenly, you have information: because if B didn't fire, but the prisoner is dead, then C must have done it. So, conditioning on a common effect (A) renders the causes of that common effect (B and C) correlated, even if they were originally independent.

How do we use this connection between causal structures and conditional independence.? Suppose we know that B and C are correlated, but become independent once we condition on A. Also suppose that we are willing to assume that all the correlations between these variables arise through their causal interaction in a DAG. Then we know that there are only three possibilities: either A is the common cause of B and C (B <- A ->C), or we have one of two chains: B -> A -> C or B <- A <- C. Thus, we can narrow down the number of causal possibilities by looking at the conditional independence relations; for instance, B -> C < -A is decisively ruled out by this situation. As a result, we can not only *test* causal models against correlational data, but we can also *search the data* for possible causal models that are consistent with them. To do this effectively, however, we need a method to analyze larger systems. As it happens there are very simple rules by which you can look at a causal graph and derive which conditional independence relations should hold in the data if that graph were true. The method to do this is called *d*-sepation. Below, we explain how *d*-separation works. First, however, we need to explain the concept of blocking.

3. Blocking

Imagine that the DAG consists of four variables: X1, X2, X3, and X4. Say we have a hypothesis on the causal relations between these variables, and want to know whether X1 and X2 would be independent given X3 if our hypothesis were correct. This is useful because it allows us to 'read off' the conditional indepencies that follow from a graph, and thus to test the causal model that the graph instantiates.

Pearl's (2000) d-separation criterion says when we have conditional independence of X1 and X2 given X3. d-separation is defined in terms of 'blocking'. There are three conditions under which a variable X3 blocks a path between two other variables, X1 and X2 (a path is any set of consecutive edges from X1 to X2; any way to get from X1 to X2 by 'traveling' the edges). First, that X3 'gets in the way' of the path from X1 to X2, by 1) being the common case of X1 and X2 or 2) being the intermediate variable in a chain from X1 to X2. Second, that conditioning on X3 does not *create* any conditional dependencies that were previously absent, which can happen when (3) X3 is the common effect of X1 and X2, or an effect of the common effect of X1 and X2 (for instance, when X4 is the common effect of X1 and X2, and X3 is an effect of X4).

Summarizing, a path between X1 and X2 is said to be blocked by variable X3 if:

1. X1 and X2 are connected by a chain in which X3 is the middle node (so here that would be X1->X3->X2 or X1<-X3<-X2), or
2. X1 and X2 are connected by a common cause, and X3 is that common cause (here: X1<-X3->X2), or
3. X1 and X2 are connected by a common effect ('collider'), but X3 is *not* that common effect, and X3 is *not* one of the effects of the common effect.

First consider conditions 1 and 2 in case there is just one path between variables X1 and X2. In both of these cases the basic idea is that *if* there is a dependence of X1 and X2, then in these cases X3 is what *has created* that dependence (either by transmitting the effect of X1 to X2 in a chain, or by being the common cause of X1 and X2). Hence 'controlling for' X3 by conditioning on it *removes* the dependence of X1 and X2. Thus, if X1 and X2 are connected by a chain or fork and X3 is the middle node in the chain or the common cause in the fork, we are done: in that case X3 blocks the path between X1 and X2.

If X1 and X2 are not connected by a chain or fork, then they must be connected by a collider (a common effect). This is because there is, by assumption, *a* path between them; so if it isn't a fork or chain, then it must be a collider (remember that we assumed that the graph is directed, so undirected connections are not admitted). In this case, X1 and X2 can still be independent given X3 (so that X3 in this sense 'blocks' the path between them), but only if X3 is not a function of the common effect.

This is because, if X3 is the common effect or a function of it, then conditioning on X3 will cause X1 and X2 to *become* dependent if they *weren't* in the original situation. This will happen as soon as X3 is the *common effect* of X1 and X2 *or* when it is one of the *effects of that common effect* (a 'child' of the common effect, e.g., when X4 is the coommon effect and X3 is an effect of X4). The reason is that if X1 and X2 were originally independent, but both can cause X3, then if one knows the value of X3 and X2, one has more information about X1 than in the unconditional situation.

For instance, suppose that soldiers X1 and X2 can both shoot the prisoner X3, and that X1 and X2 operate completely independent of each other (so there is no correlation between their shooting or not shooting; if one only knows that X1 has fired, then that carries no information about whether X2 has fired). Now, if one knows that the prisoner is dead (X3=1) and one also knows that X2 didn't fire (X1=0) then the other soldier X2 must have done it (X2=1). Thus, if one knows the state of the prisoner (dead or alive) then learning what soldier X1 did helps in determining what soldier X2 did. Thus, conditional on the soldier being dead (X3=1), there is a correlation between the variables X1 and X2; thus X1 and X2 have *become* dependent (where they were orignially independent) *by conditioning on their common effect* X3.

So, conditioning on a common effect induces dependence. Hence, if one conditions on a common effect of X1 and X2 (or on an effect of that common effect that is farther down the causal line), then X1 and X2 will become dependent. Thus, if X1 and X2 are connected by a collider, we need to make sure that fire squad scenario isn't set in motion by our conditioning on X3 (otherwise there will be dependence, and we are looking for independence here). This gives is what condition (3) does.

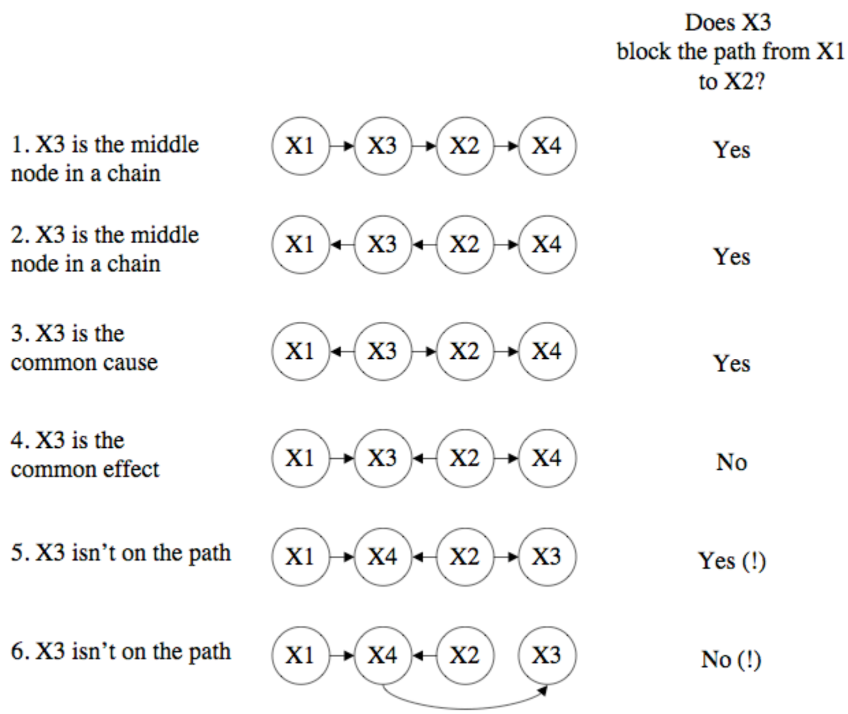
These conditions define what it means for a variable to block a given path. To recapitulate: in any path between two variables X1 and X2, there will be chains, colliders, forks, or a mixture of them. If you want to see whether X3 blocks the path, then you first check whether X3 is in it. If X3 is the middle node of a chain or the common cause in a fork, then X3 blocks the path. If X3 is the common effect in a collider, then it does not block the path. If X3 is not on the path at all, it may still block it. There is only one way in which this can happen, and this given by condition 3: If X1 and X2 are connected by a collider, and X3 is not that collider, and X3 is not one of the effects of the collider, then X3 blocks the path.

4. d-separation

In many graphs, there will be more than one path between any two variables. In this case, X3 may block one path, but not another, so that X1 and X2 could still be dependent given X3, even if X3 blocks *a* path between them. The concept of d-separation (the 'd' is from 'directional') takes care of this, by requiring that no dependence can be induced by *any* of the paths between X1 and X2 once we condition on X3. The definition of d-separation is that *a variable d-separates two other variables if it blocks all the paths between them.*

d-separation is an extremely powerful concept, because one only has to look at the graph to determine that two variables X1 and X2 are d-separated by X3; if one sees that they are, then it follows that the variables X1 and X2 are conditionally independent given X3 *in all probability distributions that are consistent with the graph*, *regardless of how the* *variables are distributed or what the functional form of the relation between is.* You may sit and pause here for a moment, and wonder how such a simple graphical criterion can have such enormous consequences; this is really amazing.

Below are some graphs that illustrate the concept of blocking and *d*-separation.



In cases 1 and 2 in the figure, X3 mediates the effect of X1 on X2 and therefore blocks the path between X1 and X2. Because this is the only path between X1 and X2, X3 d-separates them. Consequence: if you made a *separate* contingency table with X1 and X2 *for each value of* X3, then you'd find that X1 and X2 are *independent* in each of these tables. (See the handout on probability theory on Blackboard).

In case 3 in the figure, X3 is the common cause of X1 on X2 and therefore blocks the path between X1 and X2. Because this is the only path between X1 and X2, X3 d-separates them, and the same consequence as in the cases 1 and 2 follows: if you'd make a *separate* contingency table with X1 and X2 *for each value of* X3, then you'd find that X1 and X2 are *independent* in each of these tables.

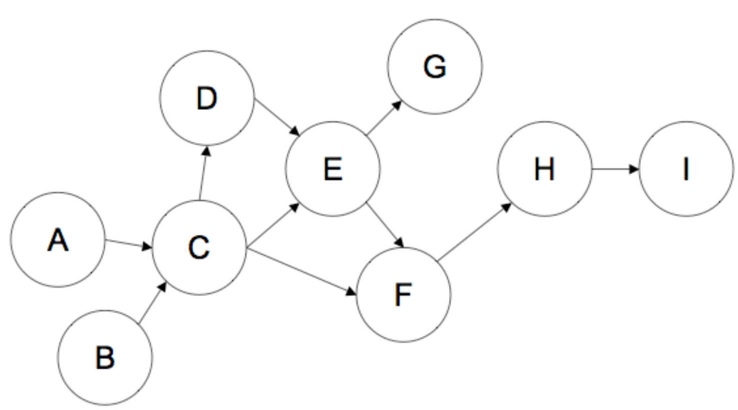
In case 4 in the figure, X3 is the common effect of X1 on X2 and therefore does *not* block the path between X1 and X2. Thus X3 does not d-separate them: if you made a *separate* contingency table with X1 and X2 *for each value of* X3, then you'd find that X1 and X2 are *not independent* in each of these tables. (Remember the story of the fire-squad: if the prisoner X3 is dead, and soldier X1 didn't shoot, then soldier X2 must have done it).

In case 5 of the figure, X3 isn't on the path between X1 and X2. However, X4 is: it is the common effect of X1 and X2. Now condition (3) comes into play. We check whether X3 is a child of the common effect X4; this is *not* the case. Thus, X3 blocks the path between X1 and X2. Because this is the only path between X1 and X2, X3 d-separates them: if you make a *separate* contingency table with X1 and X2 *for each value of* X3, then you'll find that X1 and X2 are *independent* in each of these tables. It may seem weird that a variable that isn't on the path (and in fact has nothing to with it) can 'block' that path. However, here 'blocking' just means that X1 does not give you information about X2 once you know the value of X3. In this case, X1 and X2 weren't dependent to begin with (otherwise there would have been an arrow connecting them – remember we are only considering DAGs here), and the trick here is that X3 doesn't *make* them dependent (as it does in cases 4 and 6).

In case 6 of the figure, X3 isn't on the path between X1 and X2. However, X4 is: it is the common effect of X1 and X2. Again condition (3) comes into play. We check whether X3 is a child of X4; this *is* the case. Thus, X3 does *not* block the path between X1 and X2, and therefore does *not* d-separate them: if you make a *separate* contingency table with X1 and X2 *for each value of X3*, then you'll find that X1 and X2 are *not independent* in each of these tables. The logic here is that conditioning on X3 means that you are conditioning on a consequence of the common effect. So X3 carries information about that common effect, which in turn renders X1 and X2 dependent in the same way as case 4 does. Consider again the story of the fire-squad. Suppose soldiers 1 (X1) and 2 (X2) can kill the prisoner (X4), and the prisoner will fall down if killed (X3). If we learn that the prisoner fell down (X3=down), the he was probably shot (X4=killed), and if soldier 1 didn't do it (X1=didn't shoot) then probably soldier 2 did (X2=did shoot). So information travels back from the consequence farther down the line (X3) via the common effect (X4) to render the causes of that common effect (X1 and X2) dependent.

5. Exercise

As an exercise, consider the following Directed Acyclic Graph of Figure 1 again:



1. Consider the variables A and E.

a) list all the paths that connect A and E

b) for every path between A and E, list the variables that block that path
c) which variable d-separates A and E?
d) list the conditional independence relations that your answer to (c) entails.

2. Consider the path C-D-E.
a) which variable blocks this path?
b) are C and E independent given D?
c) does the DAG contain a variable that d-separates C and E?

3. Consider the variables B and I.
a) list all the paths that connect B and I
b) which variables d-separate B and I?
c) which conditional independence relations does your answer to (b) imply?

4. Consider the variables A and B.
a) list all directed paths that connect A and B.
a) are A and B conditionally independent given C?
b) are A and B conditionally independent given D?
c) are A and B conditionally independent given I?
d) are there any variables in the graph that d-separate A and B?
e) are A and B unconditionally independent (this is the same as independence given the empty set)?

5. Consider the variables E, F, and G.
a) are G and F conditionally independent given E?
b) are G and F conditionally independent given C?
c) are G and F conditionally dependent given C?

Literature

Pearl, J. (2000, 2nd Ed .2009). Causality: Models, reasoning, and inference. Cambridge, England: Cambridge University Press.